Geometry in Entanglement Percolation

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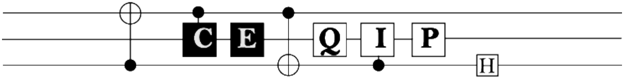
June 8, 2014 CEQIP, Znojmo, Czech Republic



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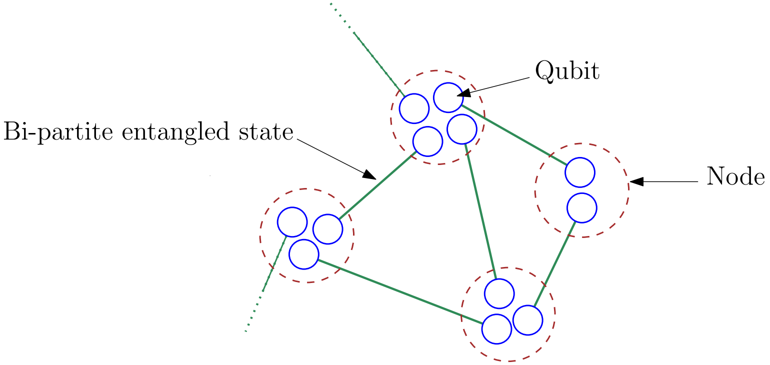


- *qubits* (two-level quantum systems) (spin $1/2$, *e.g.* photon polarization)
- Multiple qubits and classical resources at each *node* (vertex)
- *links* (edges): bi-partite entangled (pure/mixed) two-qubit states.
- Goal: entangle pairs of qubits between distant nodes
- Quantum operations local: within nodes. Classical can be global. Local Operations and Classical Communication LOCC

Technical motivation: Generalize one-dimensional networks

- Quantum Information: Entanglement is a resource for tasks: teleportation, key distribution, fault tolerant computation
  - Creating entanglement requires local interaction. Noise increases with distance. Depolarization. Absorption. Can't distribute entanglement over long distance in a single stage!
- Long range entanglement via Network of stations or nodes that store and purify a state.
  - Generalization of quantum repeater schemes. *Dür, Briegel, Cirac, Zoller, PRA 1999*
  - Nodes share partially entangled states of qubits
  - Nodes(stations)/channels, Vertices/edges, Sites/bonds
  - Quantum operations probabilistic
  - Large number of random components $\Rightarrow$ Complex Networks, Percolation, Phase transition

Goal of Entanglement Percolation

- Given a network with a specified amount of quantum and classical resources, and a specific long range entanglement task, design the optimal protocol to acheive the task.
- E.g. Optimal: Smallest amount of resources (entanglement) per link that acheives task. Or protocol that acheives task with highest probability for a given amount of resources.
- E.g. Topology of lattice(network) may be an external constraint.
- E.g. Task: entangle fixed widely separated nodes A and B.

Entanglement: Two entangled qubits

Two entangled qubits: four-dimensional Hilbert space.

Bi-partite pure state

All such states LOCC equivalent to unique state in Schmidt basis.

$$|\alpha\rangle = \sqrt{\alpha_0}\,|00\rangle + \sqrt{\alpha_1}\,|11\rangle$$

$$\alpha_0 > \alpha_1 \qquad \alpha_0 + \alpha_1 = 1 \qquad \alpha_1 \in [0, 1/2]$$

Pure, partially entangled, bipartite state

$\alpha_1 = 0$: no entanglement, $\alpha_1 = 1/2$: max. entanglement

Bell State: Singlet Conversion

Partially Entangled: $|\alpha\rangle = \sqrt{\alpha_0}\,|00\rangle + \sqrt{\alpha_1}\,|11\rangle$

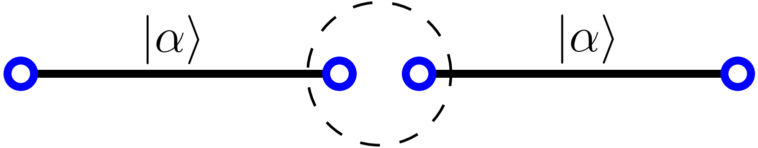
Local operations (and classical communication): qubits not allowed to interact

Maximally Entangled: $|\Psi\rangle = \frac{1}{\sqrt{2}}\,|00\rangle + \frac{1}{\sqrt{2}}\,|11\rangle$

Singlet, Bell State, Maximally Entangled State Singlet Conversion Probability $p = 2\alpha_1$, for $\alpha_0 > \alpha_1$

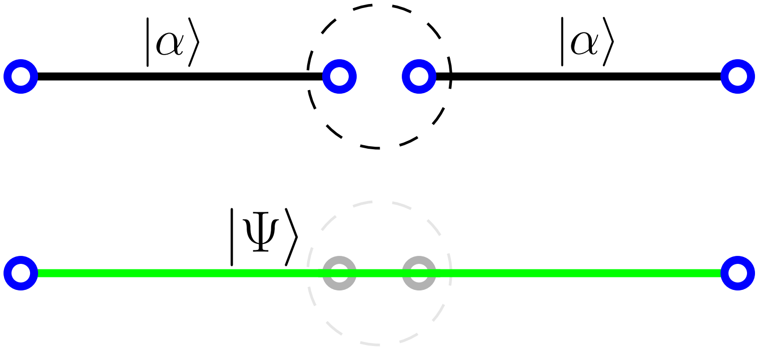
Otherwise: product state (failure)

Entanglement Swapping



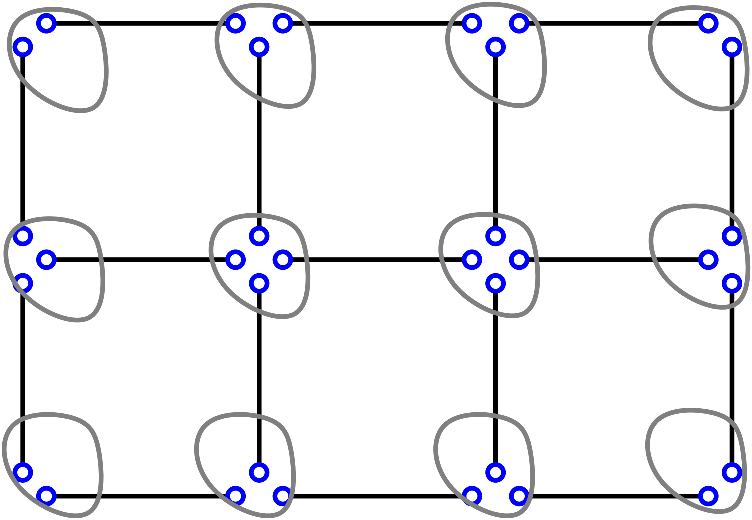
We can entangle the two outermost qubits, using only local operations and classical communication: *i.e.* without interacting outermost qubits. Using entanglement swapping.

Entanglement Swapping



Entanglement Swapping. Get Bell state with same probability as in singlet conversion $p = 2\alpha_1$! (product state otherwise) Note: if $\alpha_1 = 1/2$, then $p = 1$.

Quantum Network



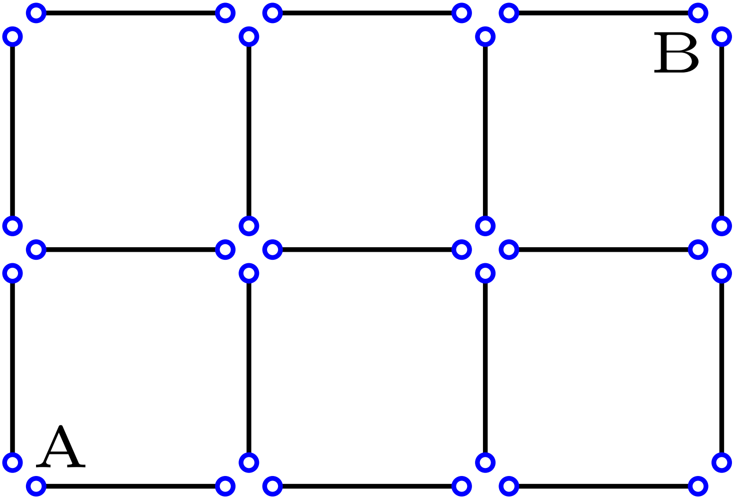
Concrete: Square lattice. Each bond is an entangled pair with amount of entanglement α_1.

Quantum Network

How to treat a network larger than two pairs. Most naive method: repeated swapping $\Rightarrow$ exponential decay. Next most naive: borrow ideas from one-dimensional quantum repeaters.

1. Attempt to put each pair in a Bell state. Here: Singlet conversion with probability of success $p = 2\alpha_1$.
2. Entanglement swappings between pairs of these Bell states. Result: New Bell state between outermost qubits, one from each of the pairs.
3. Repeat swappings, entangling ever more distant qubits.

“Classical” Entanglement Percolation



Entangle nodes A and B

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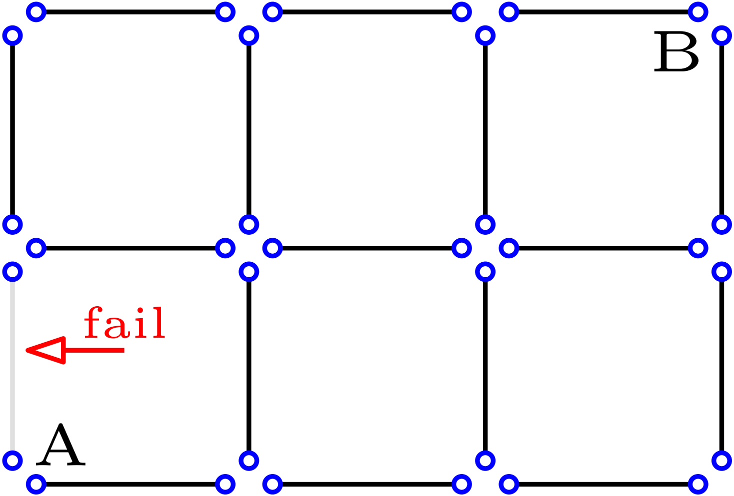

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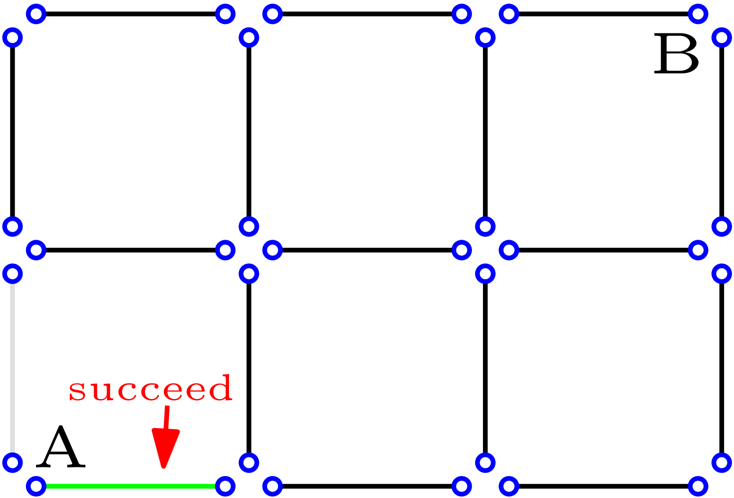

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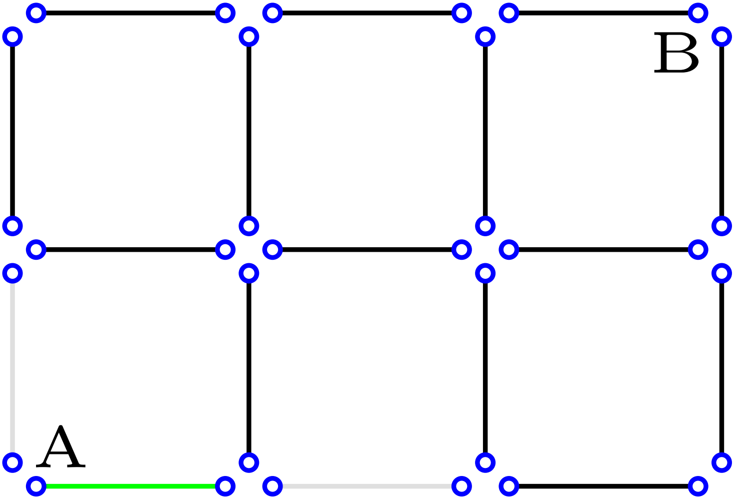

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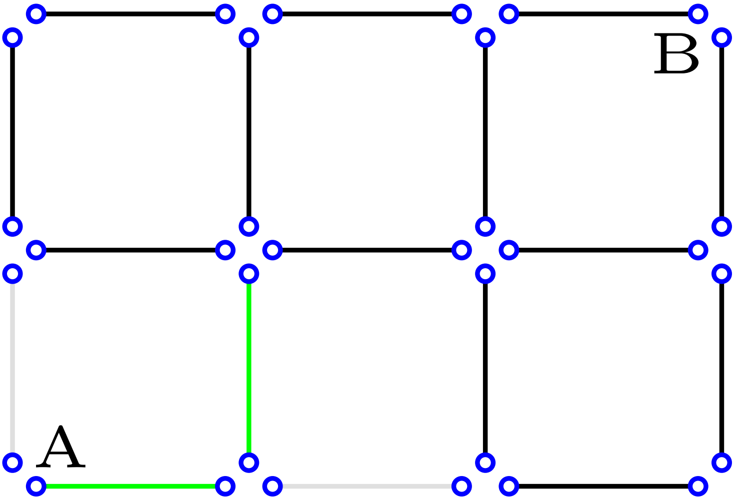

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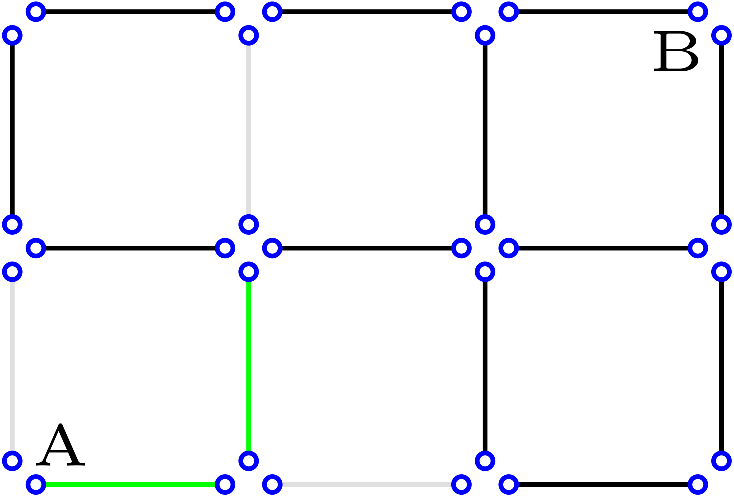

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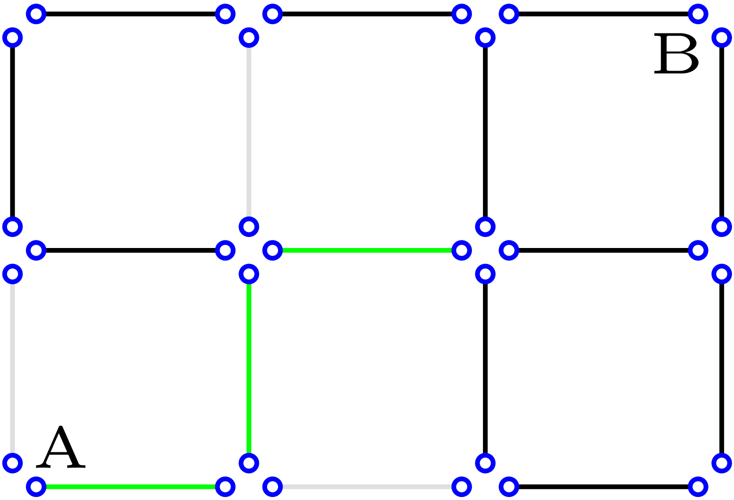

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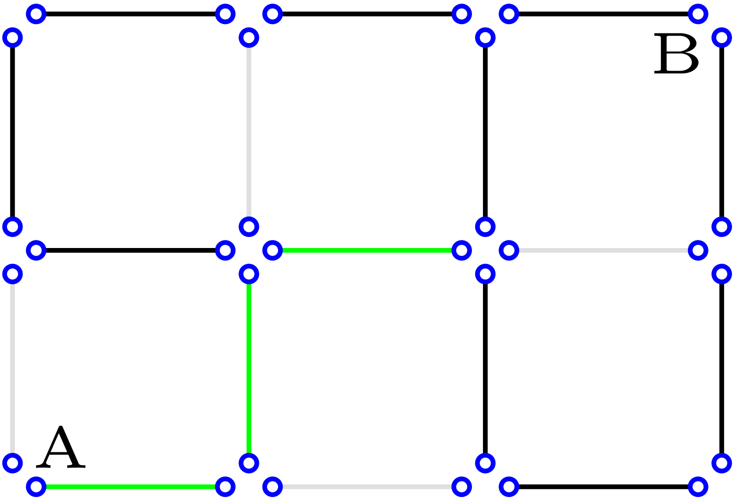

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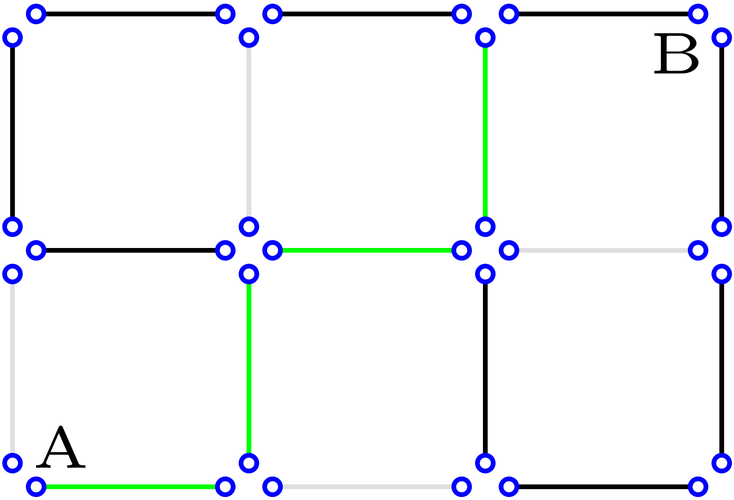

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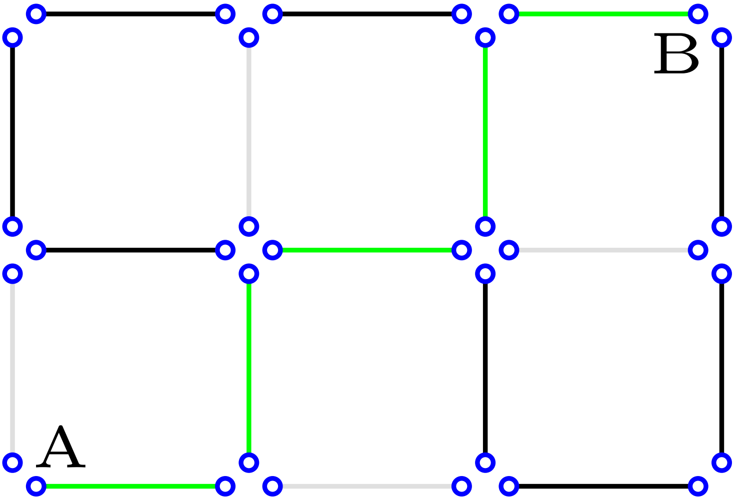

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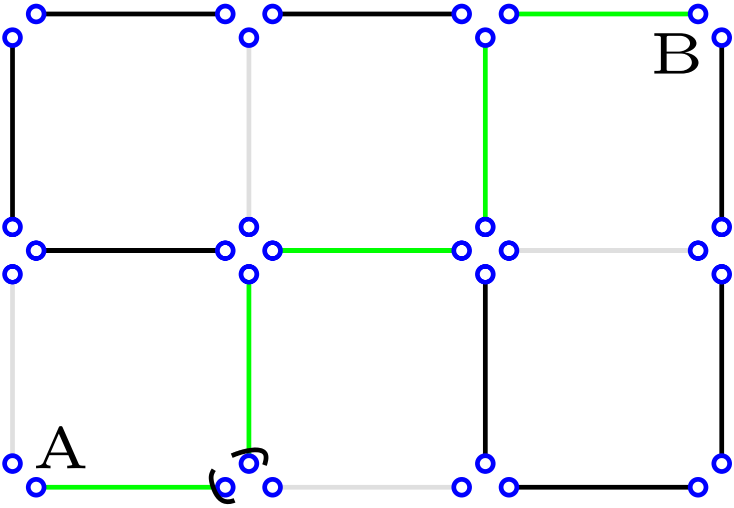

Entanglement swapping succeeds, $p = 2\alpha_1 = 1$.

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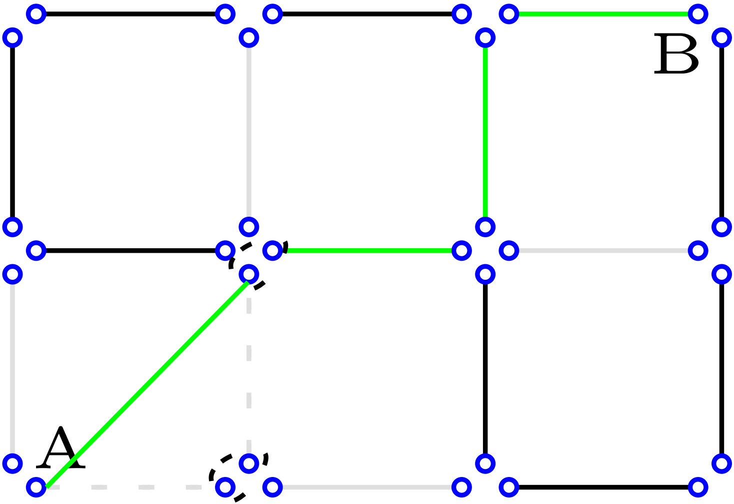

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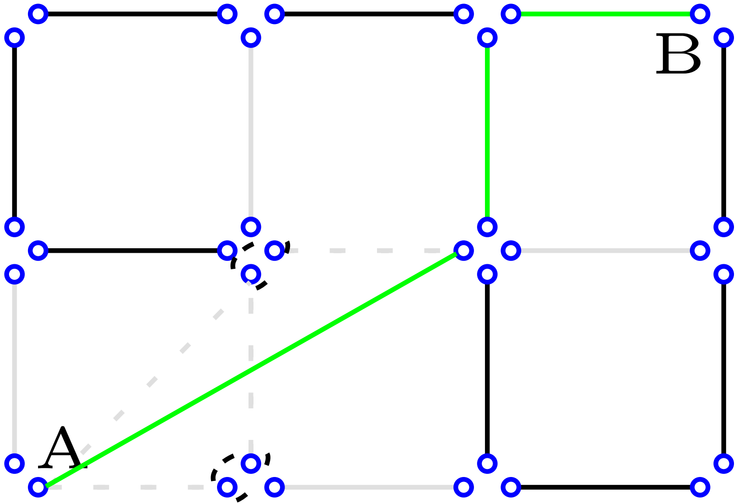

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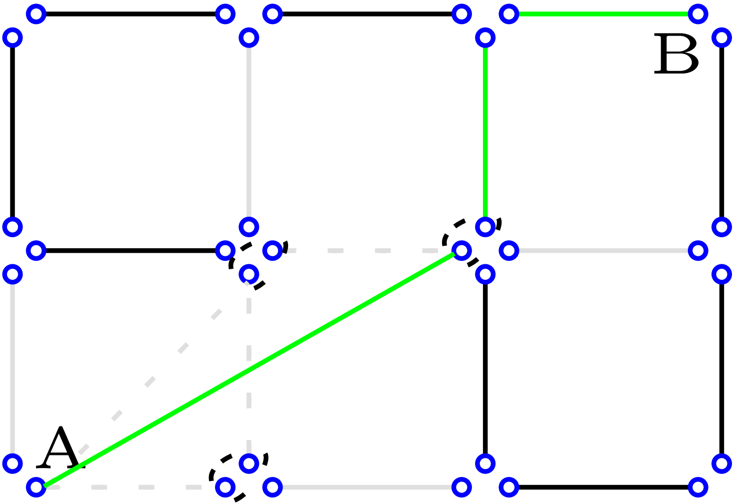

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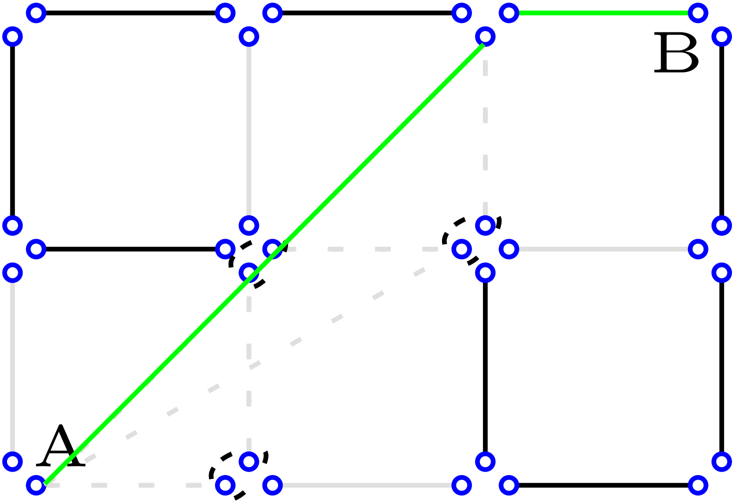

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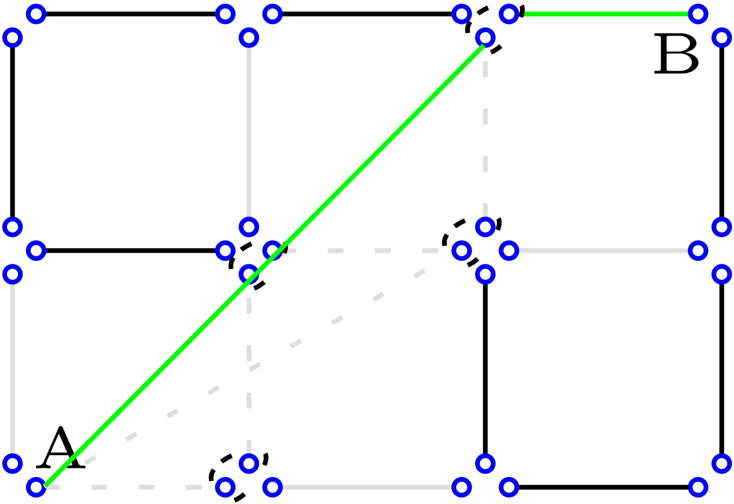

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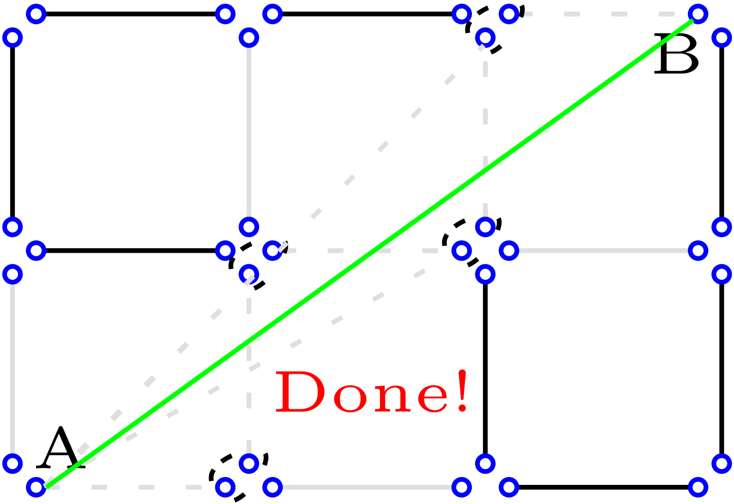

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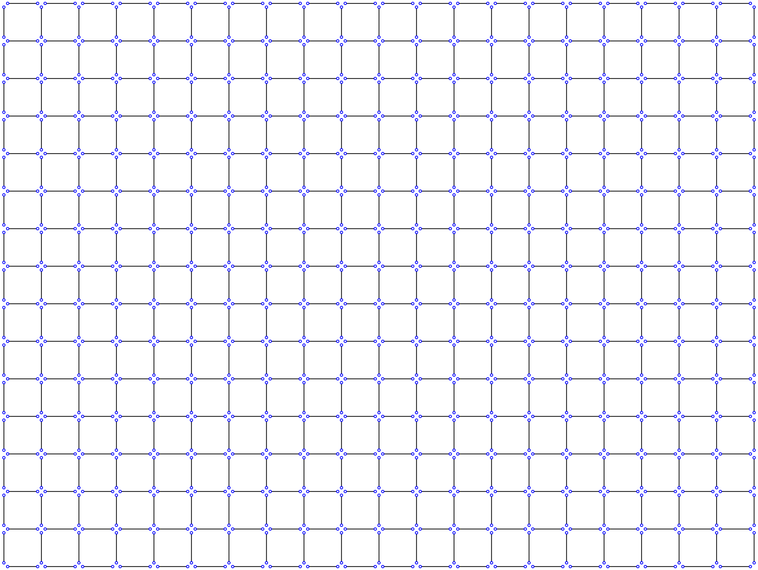
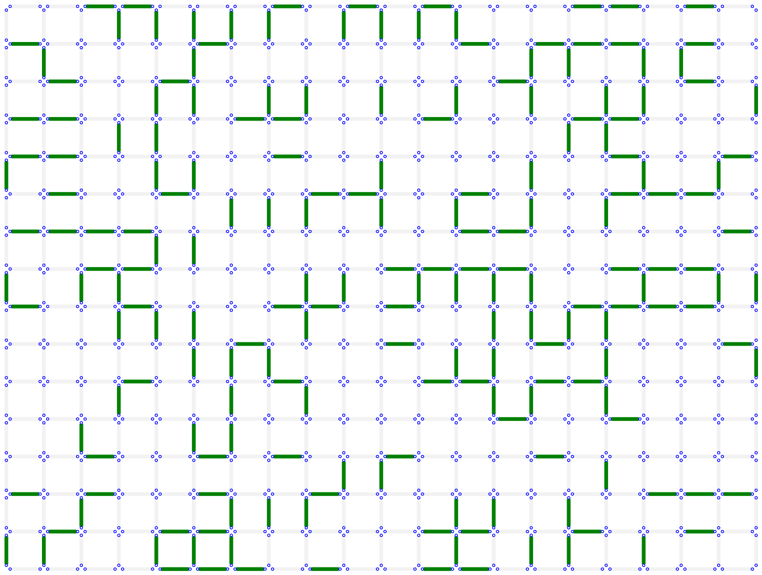
Big Network: $\alpha_1 = 0.175 \;\; p = 0.35$



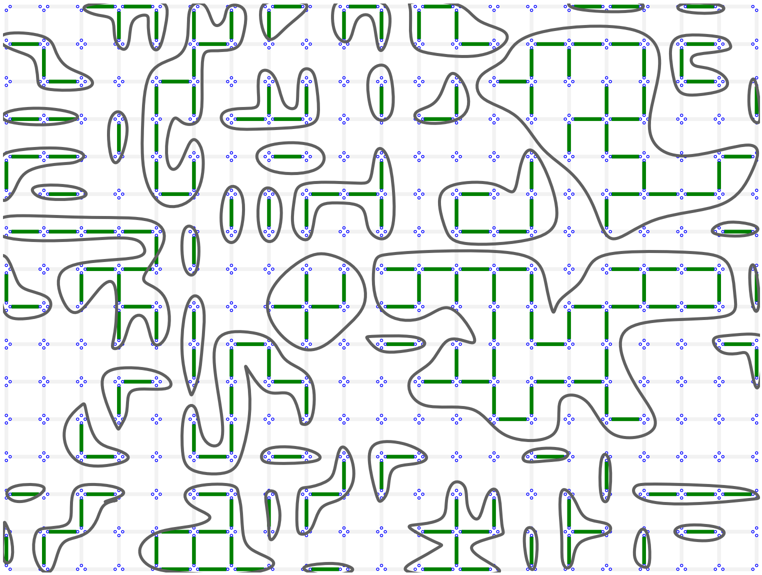
All bonds identically prepared in state $|\alpha\rangle$.

Big Network: $\alpha_1 = 0.175 \;\; p = 0.35$



Singlet conversions everywhere... Partition into clusters.

Big Network: $\alpha_1 = 0.175 \;\; p = 0.35$

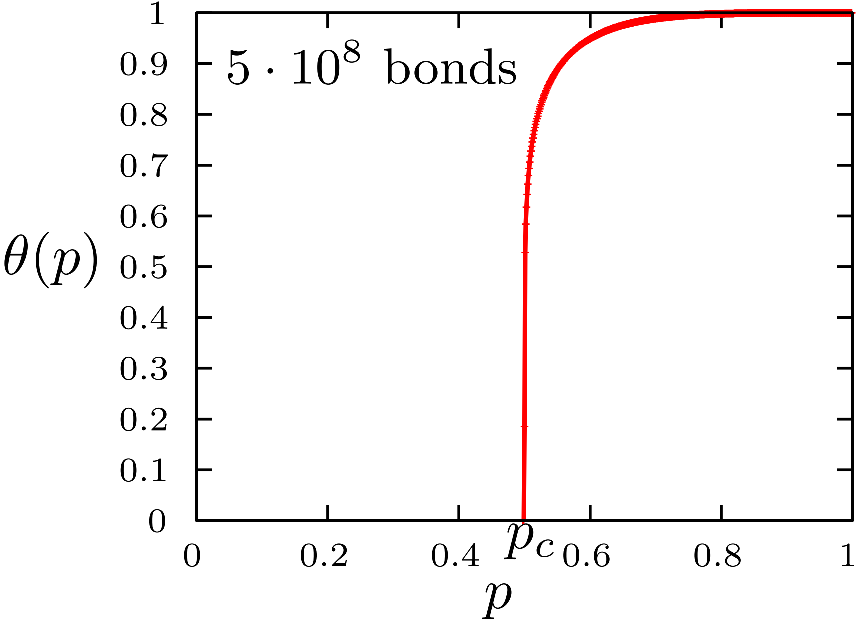


Bond percolation on square lattice. Infinite cluster iff $p > p_c = 0.5$.

Percolation theory

- Bonds are *open* (present) with probability p; or else *closed* (absent). p is called the bond density.
- For large lattices there is a threshold value of the bond density p_c. For $p > p_c$ there is a single cluster that spans the whole lattice. p_c depends on the structure of the lattice.
- In order for A and B to be connected if they are very far apart,
  1. Must have $p > p_c$
  2. Both A and B must be in the huge cluster
- Let $\theta(p) = \text{Prob}(A \text{ is in huge cluster})$
- A and B are connected with Prob = $\theta^2(p)$

Fraction of Bonds in largest cluster



Can we do better than simply swapping along a chain ?

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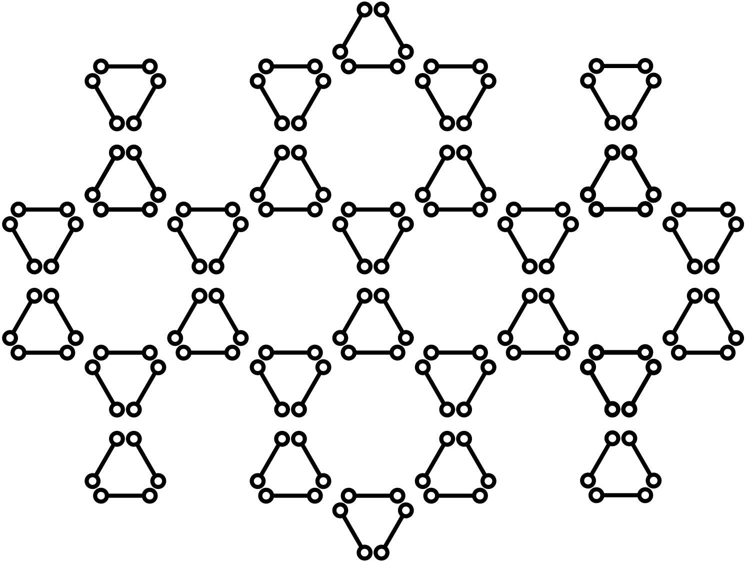
Yes. Precondition the lattice with other quantum operations.

Change local structure $\Rightarrow$ Different lattice $\Rightarrow$ Different global properties: Different percolation threshold.

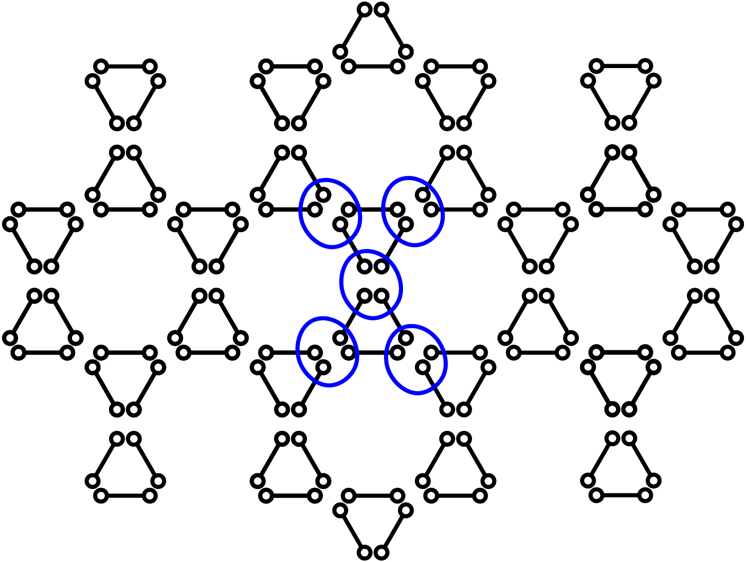
Then swap along chain.

Kagome lattice



Easiest example.

Kagome lattice



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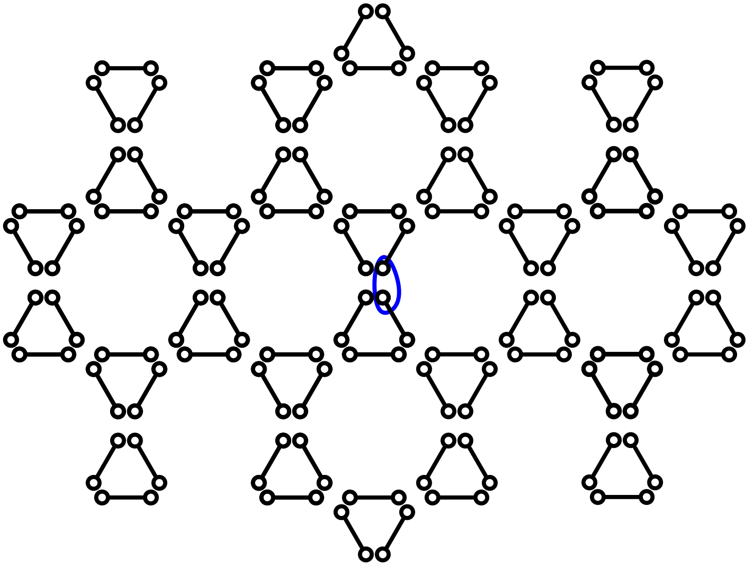
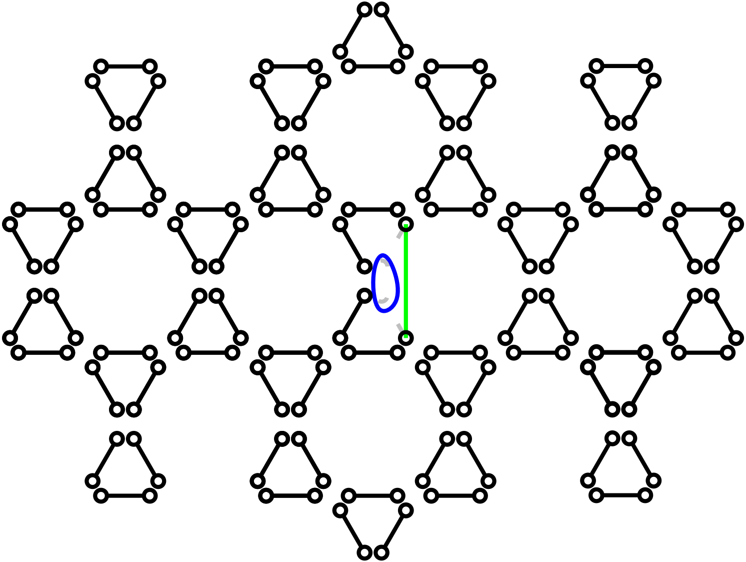


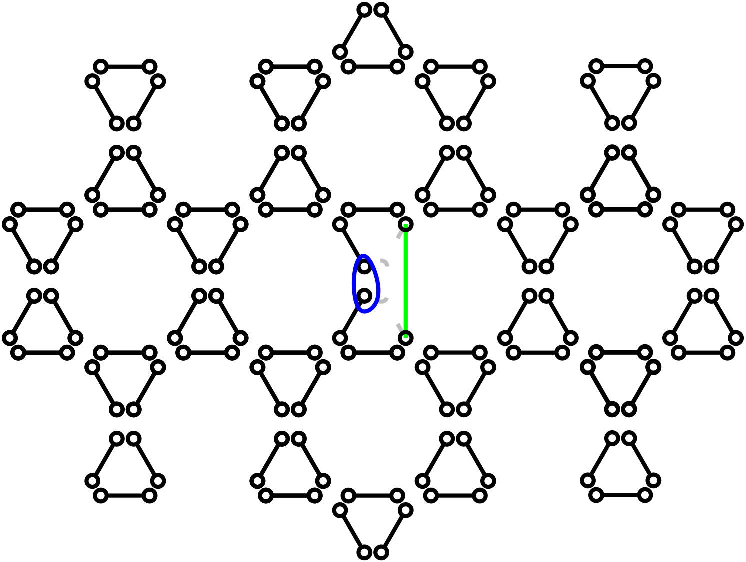
Entanglement swapping to create vertical bonds.

Kagome lattice



Vertical bonds are Bell pairs.

Kagome lattice



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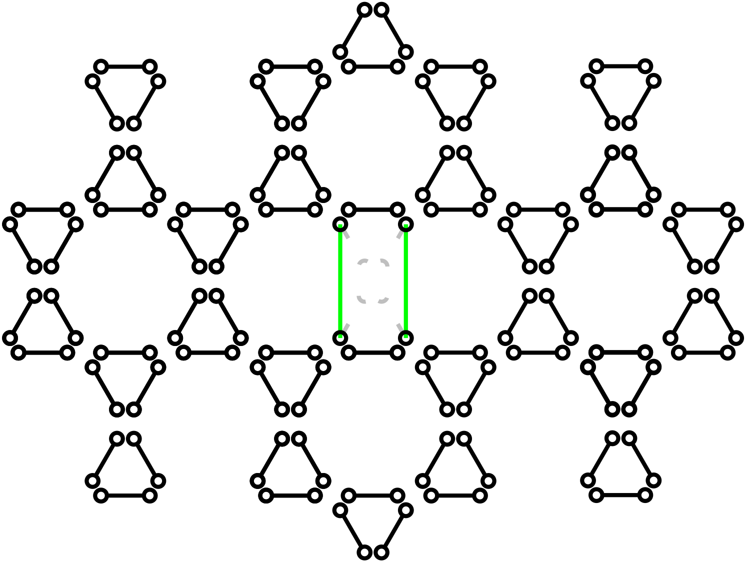
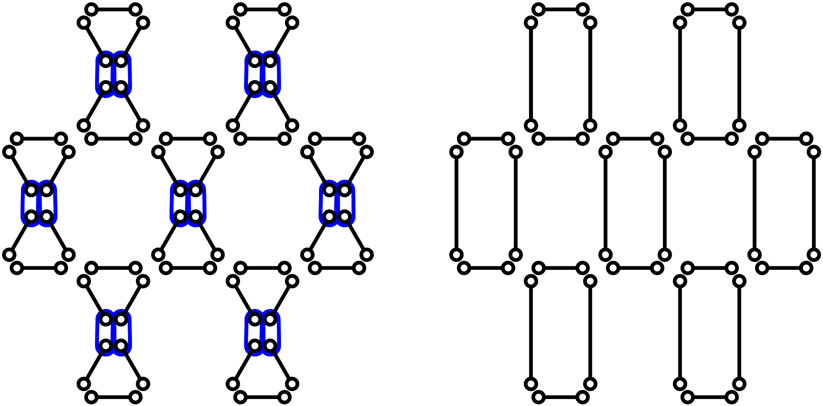


Then singlet conversion on horizontal bonds.

Convert kagome lattice to square lattice

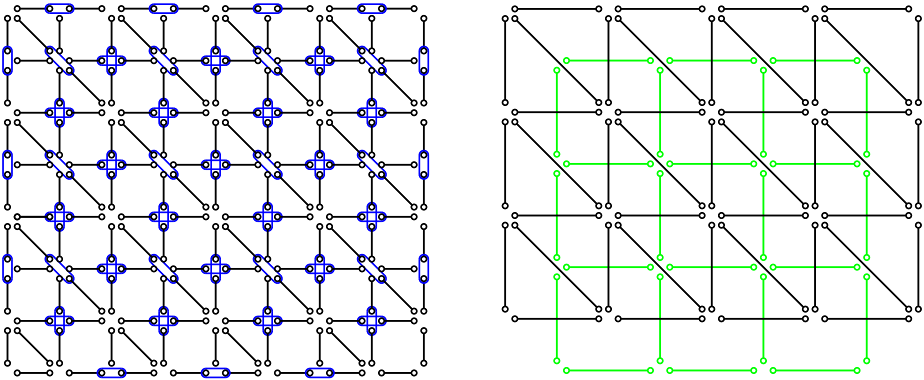


$p_c \approx 0.52$ Kagome $\qquad p_c = 0.5$ Square lattice

Acín, Cirac, Lewenstein, Nature Phys (2007)
Perseguers, Cirac, Acín, Lewenstein, Wehr, PRA (2008)

JL, Wehr, Lewenstein, PRA (2009)

Convert bowtie lattice to square and triangular



$p_c \approx 0.40$ Bowtie $\quad$ $p_c \approx 0.35$ Triangular

JL, J. Wehr, M. Lewenstein, PRA (2009)

Characterize Protocols

In all known effective preconditioning protocols:

- Local connectivity non-decreasing: Coordination number increases or remains the same.
- Global connectivity non-decreasing: Classical percolation threshold decreases.

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Characterize Protocols

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Unsurprising, but, must this always be the case?

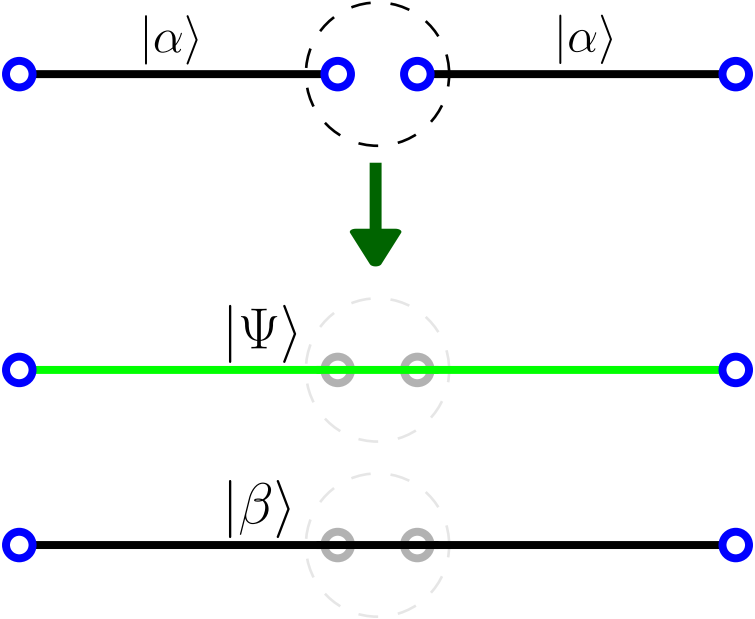
No. Counter-example: Look at swapping more closely. Only perform partial entanglement swapping.

Entanglement Swapping

Entanglement swapping procedure.

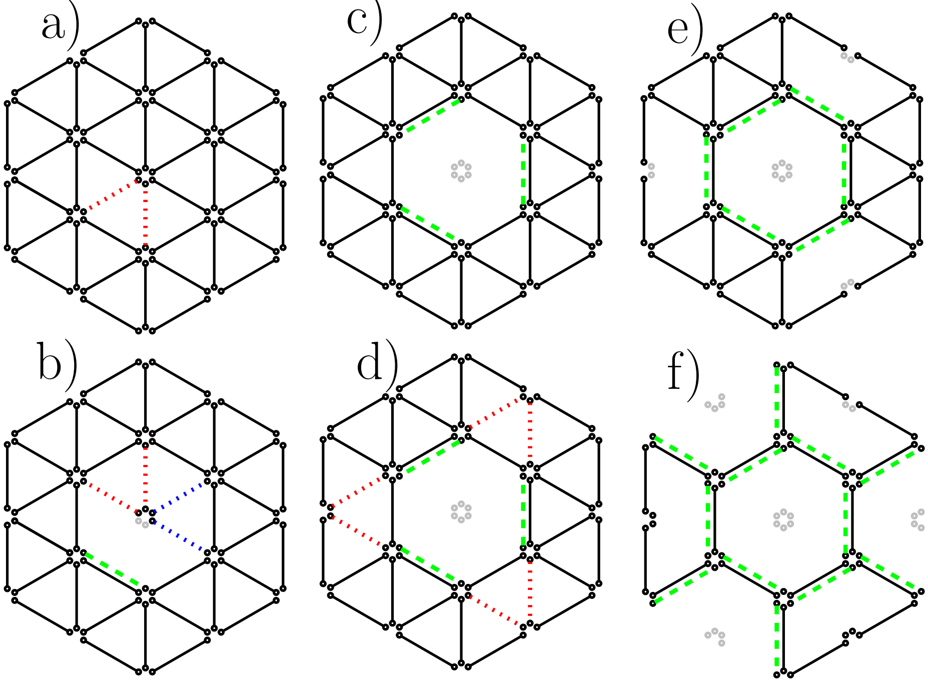
1. Measurement projecting two center qubits onto Bell basis.
2. Unitary on end qubit based on result of measurement, leaving two distinct (pairs of) states.
3. One state is partially entangled: perform SCP on it.
4. Other state is already maximally entangled.
5. Averaging over these two possibilities gives $p = 2\alpha_1$.

Entanglement Swapping



Partial swapping and distillation

Do partial swapping (only projection) on selected bonds of triangular lattice. This leaves pairs of parallel bonds in honeycomb (hexagonal) lattice, which are then distilled.



a)
b)
c)
d)
e)
f)

Partial swapping and distillation

Majorization says we can distill a singlet from the double bond pair with probability

$$p = \min\left\{1, 2\left(1 - \frac{\alpha_0^3}{\alpha_0^2 + \alpha_1^2}\right)\right\}$$

The real root of $\alpha_0^3 - \alpha_0^2 + \alpha_0 - 1/2 = 0$ is $\alpha_0^* \approx 0.647798$. Thus, if $\alpha_0 < 0.647$ then each double bond can be converted to a singlet with probability 1.

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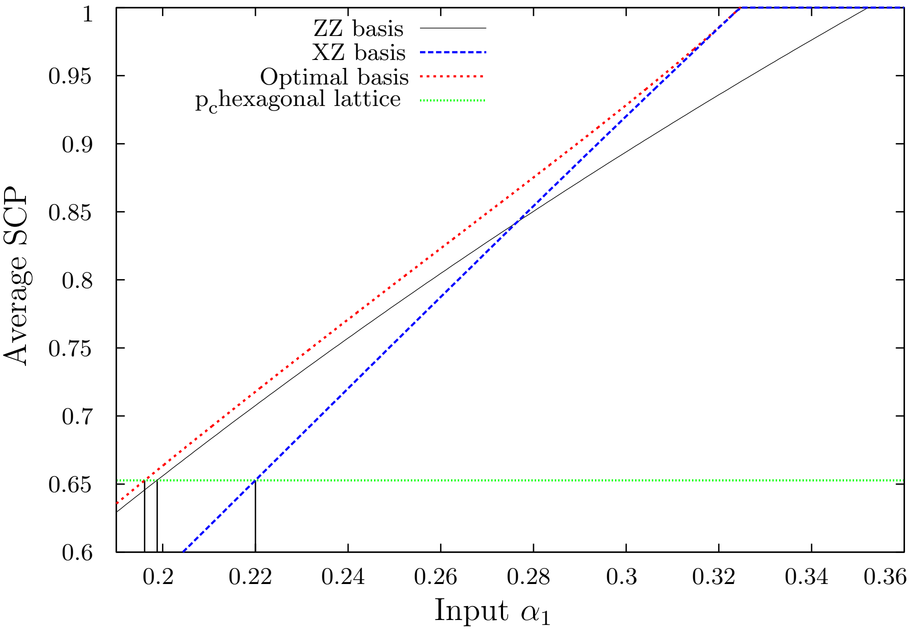
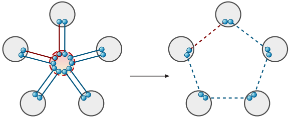
And this can be improved by optimizing the chosen Bell basis instead of taking the usual one for swapping.



Role of local and global geometry in entanglement percolation, JL, PRA (2014)

Other work

- Multipartite (GHZ) states $\Rightarrow$ percolation non-planar graphs *Perseguers, Cavalcanti, JL, Lewenstein, and Acín, PRA (2010)*
- Mixed states of rank ≤ 3 *Broadfoot, Dorner, Jaksch, PRA (2010), EPL (2009)*
- Q-star transformation on complex networks. increase entanglement distance.  *Cuquet, Calsamiglia, PRL (2009), PRA (2011)*
- Mixed states full rank, complex network. *JL, Perseguers, Lewenstein, Acín, QIC (2012)*
- Review: *Perseguers, JL, Cavalcanti, Lewenstein, Acín, Rep. Prog. Phys. 76 (2013)*

Other things . . .

- Geometry, topology, etc. often determines optimal protocol.
- Constraint on geometry of transformed lattice?
- Is there a lower bound on local entanglement density in 2-d ?
- Dynamics ?
- Coupled lattices ?